

Math 2401 Final exam review

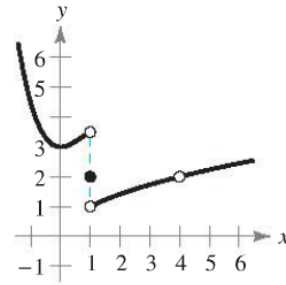
1. Use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.

$$f(1) = \underline{2}$$

$$\lim_{x \rightarrow 1} f(x) = \underline{\text{DNE}}$$

$$f(4) = \underline{\text{DNE}}$$

$$\lim_{x \rightarrow 4} f(x) = \underline{2}$$



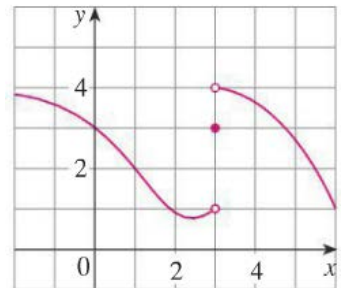
2. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, use DNE.

a. $\lim_{x \rightarrow 1} f(x)$

b. $\lim_{x \rightarrow 3^-} f(x)$

c. $\lim_{x \rightarrow 3^+} f(x)$

Solution:



a. $\lim_{x \rightarrow 1} f(x) = 2$

b. $\lim_{x \rightarrow 3^-} f(x) = 1$

c. $\lim_{x \rightarrow 3^+} f(x) = 4$

3. Find the limit of the function.

a. $\lim_{x \rightarrow 0} \frac{x^4 - 5x^2}{x^2} = \underline{-5}$

b. $\lim_{x \rightarrow -1} \frac{x^2 - 2x - 3}{x + 1} = \underline{-4}$

c. $\lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 5x + 4} = \underline{-\frac{1}{3}}$

d. $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 - 4} = \underline{\frac{1}{2}}$

4. Find the derivative of the function.

a. $f(x) = (3x^2 - 4)(2x + 5)$

$$f'(x) = 6x(2x + 5) + (3x^2 - 4)2 = 18x^2 + 30x - 8$$

b. $g(s) = \sqrt{s}(s^2 + 8)$

$$g'(s) = \frac{1}{2}s^{-\frac{1}{2}}(s^2 + 8) + s^{\frac{1}{2}}2s$$

$$= \frac{1}{2}s^{\frac{3}{2}} + 4s^{-\frac{1}{2}} + 2s^{\frac{3}{2}}$$

$$= \frac{5}{2}s^{\frac{3}{2}} + 4s^{-\frac{1}{2}} \quad \text{or} \quad \frac{5}{2}s^{\frac{3}{2}} + \frac{4\sqrt{s}}{s}$$

Math 2401 Final exam review

c. $h(t) = t^3 \cos t$

$$h'(t) = 3t^2 \cos t + t^3 (-\sin t)$$

$$= \boxed{3t^2 \cos t - t^3 \sin t}$$

d. $f(x) = \frac{\sqrt{x}}{x^2+1}$

$$f'(x) = \frac{\left(\frac{1}{2}x^{-\frac{1}{2}}(x^2+1) - x^{\frac{1}{2}}(2x)\right)}{(x^2+1)^2}$$

$$= \frac{\frac{1}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}} - 2x^{\frac{3}{2}}}{(x^2+1)^2}$$

$$= \frac{\left(-\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)}{(x^2+1)^2}$$

$$= \boxed{\frac{x^{\frac{1}{2}} - 3x^{\frac{5}{2}}}{2x(x^2+1)^2}}$$

e. $g(x) = \frac{\sin x}{x^2}$

$$g'(x) = \frac{x^2 \cos x - 2x \sin x}{x^4}$$

$$= \boxed{\frac{x \cos x - 2 \sin x}{x^3}}$$

5. Find $f'(x)$ and find the tangent line of $f(x)$ at the giving point c .

a. $f(x) = (x^2 + 4x)(3x^2 + 2x - 5)$, $c = 0$

$$f'(x) = (2x + 4)(3x^2 + 2x - 5) + (x^2 + 4x)(6x + 2)$$

$$= \boxed{12x^3 + 42x^2 + 6x - 20}$$

Tangent line: $y = -20x$

b. $f(x) = x \cos(x)$, $c = \frac{\pi}{4}$

$$f'(x) = \cos x + x(-\sin x) = \cos x - x \sin x$$

Tangent line: $y = \frac{4\sqrt{2} - \pi\sqrt{2}}{8}x + \frac{\pi^2\sqrt{2}}{32}$

c. $f(x) = \frac{x-4}{x+4}$, $c = 3$

$$f'(x) = \frac{8}{(x+4)^2}$$

Tangent line: $y = \frac{8}{49}x - \frac{31}{49}$

Math 2401 Final exam review

6. Find the derivative of the trigonometric function.

a. $f(t) = t^2 \sin t$

$$f'(t) = 2t \sin t + t^2 \cos t$$

b. $f(\theta) = (\theta^2 + 1) \cos \theta$

$$\begin{aligned} f'(\theta) &= 2\theta \cos \theta - (\theta^2 + 1) \sin \theta \\ &= 2\theta \cos \theta - \theta^2 \sin \theta - \sin \theta \end{aligned}$$

c. $y = \frac{\sec x}{x}$

$$\frac{dy}{dx} = \frac{\tan x \sec x - \sec x}{x^2}$$

d. $f(x) = x^2 \tan x$

$$f'(x) = 2x \tan x + x^2 \sec^2 x$$

e. $g(x) = \frac{\sin \theta}{1 - \cos \theta}$

$$g'(x) = \frac{\cos \theta - 1}{(1 - \cos \theta)^2}$$

7. Find the second derivative of the function.

a. $f(x) = x^4 + 2x^3 - 3x^2 - x$

$$f''(x) = 12x^2 + 12x - 6$$

b. $f(x) = \sec x$

$$f''(x) = \sec^3 x + \tan^2 x \sec x$$

8. Find the derivative of the following functions.

a. $y = \sqrt{4 - 3x^2}$

b. $f(x) = \sqrt{x^2 - 4x + 2}$

$$\frac{dy}{dx} = -3x(4 - 3x^2)^{-\frac{1}{2}}$$

$$f'(x) = (x - 2)(x^2 - 4x + 2)^{-\frac{1}{2}}$$

c. $y = \sin(2x) \cos(2x)$

$$y' = 2 \cos^2 2x - 2 \sin^2 2x$$

d. $y = x^2 \sec x^2$

$$y' = 2x \sec x^2 + 2x^3 \tan x^2 \sec x^2$$

Math 2401 Final exam review

e. $y = \sin(\pi x)^2$

$$y' = 2\pi^2 x \cos(\pi x)^2$$

f. $h(x) = 5 \tan(3x)$

$$h'(x) = 15 \sec^2(3x)$$

9. Use implicit differentiation to find y' and evaluate y' at the indicated point.

a. $y^2 - y - 4x = 0$; (0,1)

$$y'|_{(0,1)} = 4.$$

b. $3xy - 2x - 2 = 0$; (2,1)

$$y'|_{(2,1)} = -\frac{1}{6}$$

c. $2xy + y + 2 = 0$; (-1,2)

$$y'|_{(-1,2)} = 4$$

10. For the given function $f(x) = (x - 1)^2 (x + 3)$

a. Find the critical numbers.

$$c = 1, -\frac{5}{3}$$

b. Find the open intervals on which the function is increasing or decreasing.

Interval	$(-\infty, -\frac{5}{3})$	$(-\frac{5}{3}, 1)$	$(1, \infty)$
Test point	-3	0	2
$f'(x)$ +/- at the test point	+	-	+
Increasing or decreasing	Increasing	Decreasing	increasing

c. Find the inflection points.

$$\text{Inflection point } \left(-\frac{1}{3}, \frac{128}{27}\right)$$

d. Find all relative extrema by first derivative test where applicable.

Local minimum at $x = 1$ at point (1, 0).

Math 2401 Final exam review

Local maximum at $x = -\frac{5}{3}$ at point $(-\frac{5}{3}, \frac{256}{27})$.

e. Determine the open intervals on which the function is concave upward or concave downward.

Concave upward at $(-\frac{1}{3}, \infty)$.

Concave downward at $(-\infty, -\frac{1}{3})$.

11. For the given function $f(x) = 4 - x - 3x^4$

a. Find the critical numbers.

$$c = -\frac{1}{\sqrt[3]{12}}$$

b. Find the open intervals on which the function is increasing or decreasing.

Increasing interval $(-\infty, -\frac{1}{\sqrt[3]{12}})$.

Decreasing interval $(-\frac{1}{\sqrt[3]{12}}, \infty)$.

c. Find the inflection points.

Inflection point (0,4).

d. Find all relative extrema by first derivative test where applicable.

Absolute maximum at $x = -\frac{1}{\sqrt[3]{12}}$ at point $(-\frac{1}{\sqrt[3]{12}}, \frac{3+16\sqrt[3]{12}}{4\sqrt[3]{12}})$.

12. Find the rate of change $\frac{ds}{dt}$ of the function $S = x^3 + 4y$ when $x = 5, y = -6, \frac{dx}{dt} = -1$, and $\frac{dy}{dt} = 2$.

Solution:

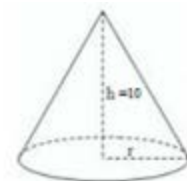
$$\begin{aligned} \frac{ds}{dt} &= 3x^2 \frac{dx}{dt} + 4 \frac{dy}{dt} \\ &= 3 \cdot 5^2 \cdot (-1) + 4 \cdot 2 = \boxed{-67} \end{aligned}$$

13. A rectangular box has base width twice the size of its base length. The volume of this box is 72 cubic units. Find the dimensions that will minimize surface area.

Length: 3, width: 6, height: 4.

14. The radius r of a right circular cone is increasing at a rate of 6 inches per minute, while the height h of the cone remains constant at 10 inches. Find the rate of change of the volume V with respect to the time t , when $r = 12$ inches. The volume of a right circular cone is $V = \frac{\pi}{3} r^2 h$.

$$\frac{dV}{dt} = \frac{\pi}{3} \cdot 2r \frac{dr}{dt} h = \frac{\pi}{3} (2)(12)(6)(10) = 480\pi.$$



Math 2401 Final exam review

15. Find all horizontal and vertical asymptotes.

a. $f(x) = \frac{x}{x^2-4}$

Horizontal asymptotes:

$y = 0$.

Vertical asymptotes:

$x = -2, x = 2$

b. $f(x) = \frac{x^2}{x-3}$

Horizontal asymptotes:

DNE

Vertical asymptotes:

$x = 3$

16. Find an indefinite integral.

a. $\int (1 + 6x)^4 dx$

$= \frac{1}{30} (1 + 6x)^5 + c$

c. $\int t^3 \sqrt{2t^4 + 4} dt$

$= \frac{1}{4} (2t^4 + 4)^{\frac{1}{2}} + c$

b. $\int x(x^2 - 9)^3 dx$

$= \frac{1}{8} (x^2 - 9)^4 + c$

d. $\int \frac{x^2}{(4x^3-9)^3} dx$

$= -\frac{1}{24(4x^3-9)^2} + c$

17. Evaluate the definite integral

a. $\int_0^4 \frac{1}{\sqrt{2x+1}} dx$

$= \sqrt{2x+1} \Big|_1^4$

$= 2$

b. $\int_1^2 2x^2 \sqrt{x^3 + 1} dx$

$= \frac{4}{9} (x^3 + 1)^{\frac{3}{2}} \Big|_1^2$

$= 12 - \frac{8}{9} \sqrt{2}$

18. Evaluate the integral using the properties of even and odd functions.

a. $\int_{-2}^2 x^2(x^2 + 1) dx$

$= \frac{272}{15}$

b. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \cos x dx$

$= 0$.

19. Differentiate the function.

a. $\int_0^x t^2 \sin t dt$

$F(x) = \int_0^x t^2 \sin t dt$

$F'(x) = x^2 \sin x$

b. $\int_2^{x^2} t + 5 dt$

$F(x) = \int_2^{x^2} t + 5 dt$

$F'(x) = x^2 + 5$

Math 2401 Final exam review

20. Find the area of region $A1$ and area of region $A2$, between the curves $y = x + 1$ and $y =$

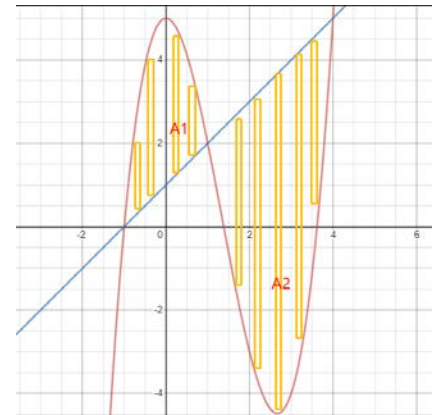
$$x^3 - 4x^2 + 5$$

Find the intersection of two curves set $x + 1 = x^3 - 4x^2 + 5$

Obtain $x = -1, 1, 4$.

$$A1 = \int_{-1}^1 (x^3 - 4x^2 + 5) - (x + 1) dx = \frac{16}{3}$$

$$A2 = \int_1^4 (x + 1) - (x^3 - 4x^2 + 5) dx = \frac{63}{4}$$



21. Find the area bounded by the graphs of the equations.

$$f = \cos^2 x, g = \sin x \cos x, x = -\frac{\pi}{2}, \text{ and } x = \frac{\pi}{4}.$$

$$\text{Area} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} \cos^2 x - \sin x \cos x dx = \frac{3\pi+4}{8}$$

